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Conformal Supergravity from the AdS/CFT Correspondence

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Abstract

Construction of a five dimensional conformal supergravity ($D = 5$ CSG) is attempted by applying the AdS/CFT correspondence to the F(4) AdS supergravity in six dimensions. As a first step, local transformation laws of $D = 5$ CSG have been established, from which the Weyl weights of the various fields in $D = 5$ can be predicted.

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1. Introduction

The main usage of the AdS/CFT conjecture has been concentrated on the study of the strong coupling limit of conformal field theories on the boundary [1], [2], [3], [4], since there is a one-to-one correspondence between the AdS supergravity fields in the bulk and the operators belonging to the representation of the superconformal group on the boundary.

Our studies have, however, been focused on the local symmetries, in which the local symmetries in the bulk become those on the boundary by applying a certain gauge fixing condition to the AdS supergravity fields. We obtain multiplets of the conformal supergravity (CSG) on the boundary [5], [6], [7], by taking the near boundary limit of the AdS supergravity in the bulk. These CSG fields play the roles of sources to the corresponding operators of a certain SCFT. Bulk local symmetries have some relations to the boundary global symmetries for the SCFT, though they are not related directly. For example, in the case of three dimensional bulk, we can reproduce the central charge of two dimensional CFT, which is the coefficient of the CSG local anomalies from the AdS supergravity Lagrangian [8].

Let us summarize our general strategy briefly. Consider a $(p+2)$-dimensional AdS supergravity, and let r denote the direction normal to the boundary. The AdS supergravity has local symmetries under the general coordinate, the local Lorentz, the supersymmetry and the gauge transformations, if the theory is gauged. Here we emphasize that there is neither Weyl nor super Weyl transformations in the bulk.

These symmetries are induced on the boundary from the general coordinate transformation in the r-direction and the supersymmetry transformation, respectively, and they form the superconformal transformations. Together with other local symmetries such as $p+1$-dimensional general coordinate transformations, they become local symmetries of the CSG.

Here, the important point is the choice of gauge fixing condition in the bulk. Our method is essentially independent of the dimension of the AdS space and the type of the supergravity. The difference of the dimensions imposes only the different conditions on fermion fields.

In the previous papers, according to the original AdS/CFT conjecture, we treated the type IIB string/M-theoretical cases. In this paper, we will study the six dimensional massive type IIA AdS supergravity and attempt to derive the corresponding

CSG from it.

In the analysis we have done previously, we understand that the one dimensionally reduced CSG local symmetries are obtained from the original AdS supergravity with the same gauge and supersymmetry. So we can expect that the six dimensional local symmetries of the AdS supergravity induce those of five dimensional CSG on the boundary. Unfortunately, few odd dimensional CSGs have been known so far.

For that reason, our trial means to construct a certain type of CSG from the AdS supergravity. Our challenge may seem to be brave, because we have a remarkable difference to overcome in the six dimensional case. We can not define the chirality nor impose the Weyl condition on the boundary, since we are considering the spacetime of five dimensions. The situation is in contrast to the two dimensional and six dimensional cases. Therefore, we will introduce a new projective gamma matrix $\tilde{\Gamma}$ instead of the chirality operator and get over the difference.

Our convention of the metric is $\eta_{AB} = \mathrm{diag}(-1, 1, \cdots, 1)$, where $A, B, \cdots = 0, 1, \cdots, 5$ are six dimensional local Lorentz indices. The 16×16 gamma matrices Γ^A are defined by

$$\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}, \tag{1}$$

and we define

$$\Gamma_7 = \Gamma^0 \cdots \Gamma^5. \tag{2}$$

2. $D = 6$, F(4) AdS supergravity

Let us start with the relationship between the supergravity and D-brane configuration. Although $D = 6$, F(4) AdS supergravity [9] has been constructed in 1980's, it was taken notice again in the D4-D8 system where its brane solution has known to become the AdS_6 metric [10]. The warped compactification of the type IIA massive supergravity gives the solution for the sphere times the six dimensional F(4) AdS supergravity [11], [12]. On the boundary, a dual field theory is coupled to the five dimensional CSG derived from the AdS_6 supergravity, and the operators of the dual field theory have been already discussed [13].

Let us go back to the original supergravity story. F(4) is the supergroup whose bosonic subalgebra is SO(2,5) $\times$ SU(2). The field contents are a six dimensional vielvein, $e_M{}^A$, three gauge vectors A^I_M of SU(2) gauge group, an antisymmetric tensor field B_{MN}, a scalar field ϕ, four Rarita-Schwinger fields ψ_{Mi}, and four spin-$\frac{1}{2}$ fields χ_i. Here $M, N, \cdots (= 0, 1, 2, \cdots, 5)$ denote the world indices, $I, J, \cdots (= 1, 2, 3)$ the vector indices of the gauge group SU(2) and $i, j, \cdots (= 1, 2)$ the spinor indices of SU(2), respectively. The spinor fields satisfy the SU(2) symplectic Majorana condition.

The covariant derivative for arbitrary spinor ϵ_i is defined as follows:

$$\mathcal{D}_M \epsilon_i \equiv (\partial_M + \frac{1}{4}\omega_M{}^{AB}\Gamma_{AB})\epsilon_i - i\frac{1}{2} g A^I_M (\sigma^I)_i{}^j \epsilon_j, \tag{3}$$

where g is the coupling constant, and σ^I are the Pauli matrices. The local transformations are the six dimensional general coordinate, the local Lorentz, the super and SU(2) gauge transformations. With these local symmetries, the Lagrangian of $D = 6$ F(4) AdS supergravity [9] stands up to 4-fermi terms as

$$\begin{aligned}
e^{-1}L = & -\frac{1}{4}R - i\frac{1}{2}\bar{\psi}^i_M \Gamma^{MNP}\mathcal{D}_N \psi_{Mi} - i\frac{1}{2}\bar{\chi}^i \Gamma^M \mathcal{D}_M \chi_i - \frac{1}{2}(\mathcal{D}^M\phi)(\mathcal{D}_M\phi) \\
& -\frac{1}{4}\,\mathrm{e}^{-2\sqrt{\frac{1}{2}}\phi}\left(m^2 B_{MN}B^{MN} + F^I_{MN}F^{MNI}\right) - \frac{3}{4}\,\mathrm{e}^{4\sqrt{\frac{1}{2}}\phi}\partial_{[M}B_{NP]}\partial^{[M}B^{NP]} \\
& -\frac{1}{8}e^{MNPQRS}B_{MN}\left(\frac{1}{3}m^2 B_{PQ}B_{RS} + F^I_{PQ}F^I_{RS}\right) \\
& +\frac{1}{4\sqrt{2}}\,\mathrm{e}^{-\sqrt{\frac{1}{2}}\phi}\bar{\psi}^{Pi}\Gamma_{[P}\Gamma^{MN}\Gamma_{Q]}\left(mB_{MN}\delta^j_i - i\Gamma_7 F^I_{MN}(\sigma^I)_i{}^j\right)\psi^Q_j \\
& -i\frac{1}{4\sqrt{2}}\,\mathrm{e}^{-\sqrt{\frac{1}{2}}\phi}\bar{\psi}^{Pi}\Gamma^{MN}\Gamma_P\left(mB_{MN}\delta^i_j + i\Gamma_7 F^I_{MN}(\sigma^I)_i{}^j\right)\chi_j \\
& +\frac{1}{8\sqrt{2}}\bar{\chi}^i\Gamma^{MN}\left(mB_{MN}\delta^i_j + i\Gamma_7 F^I_{MN}(\sigma^I)_i{}^j\right)\chi_j \\
& -\frac{1}{\sqrt{2}}(\partial_N\phi)\bar{\psi}^i_M\Gamma^N\Gamma^M\chi_i - i\frac{1}{8}\,\mathrm{e}^{2\sqrt{\frac{1}{2}}\phi}\bar{\psi}^{Mi}\Gamma_{[P}\Gamma_7\Gamma^{QRS}\partial_{[Q}B_{RS]}\Gamma_{N]}\psi^N_i \\
& +\frac{1}{4}\,\mathrm{e}^{2\sqrt{\frac{1}{2}}\phi}\bar{\psi}^{Pi}\Gamma_7\Gamma^{QRS}\partial_{[Q}B_{RS]}\Gamma_P\chi_i + i\frac{1}{8}\,\mathrm{e}^{2\sqrt{\frac{1}{2}}\phi}\bar{\psi}^{Mi}\Gamma_{[P}\Gamma_7\Gamma^{QRS}\partial_{[Q}B_{RS]}\chi_i \\
& +\frac{1}{4\sqrt{2}}\left(g\,\mathrm{e}^{\sqrt{\frac{1}{2}}\phi} + m\,\mathrm{e}^{-3\sqrt{\frac{1}{2}}\phi}\right)\bar{\psi}^i_M\Gamma^{MN}\Gamma_7\psi_{Ni} \\
& -i\frac{1}{4\sqrt{2}}\left(g\,\mathrm{e}^{\sqrt{\frac{1}{2}}\phi} - 3m\,\mathrm{e}^{-3\sqrt{\frac{1}{2}}\phi}\right)\bar{\psi}^i{}_M\Gamma^M\Gamma_7\chi_i
\end{aligned}$$

$$
+\frac{1}{8\sqrt{2}}\left(g\,\mathrm{e}^{\sqrt{\frac{1}{2}}\phi}-7m\,\mathrm{e}^{-3\sqrt{\frac{1}{2}}\phi}\right)\bar{\chi}^i\Gamma_7\chi_i
$$

$$
+\frac{1}{8}\left(g^2\,\mathrm{e}^{2\sqrt{\frac{1}{2}}\phi}+4gm\,\mathrm{e}^{-2\sqrt{\frac{1}{2}}\phi}-m^2\,\mathrm{e}^{-6\sqrt{\frac{1}{2}}\phi}\right), \tag{4}
$$

where m is the parameter, and we have introduced e^{MNPQRS} by

$$
e_{MNPQRS}=\Gamma_{MNPQRS}\Gamma_7. \tag{5}
$$

The SU(2) vector gauge field strength is defined by

$$
F^I_{MN}\equiv 2\partial_{[M}A^I_{N]}+g\epsilon^{IJK}A_M{}^JA_N{}^K. \tag{6}
$$

The local supertransformations are up to 2-fermi terms

$$
\begin{aligned}
\delta_Q e_M{}^A &\equiv -i\bar{\psi}^i_M\Gamma^A\epsilon_i\\
\delta_Q A^I_M &\equiv i\frac{1}{\sqrt{2}}\,\mathrm{e}^{\sqrt{\frac{1}{2}}\phi}(\sigma^I)_i{}^j\left(\bar{\psi}^i_M\Gamma_7\epsilon_j+\frac{1}{2}\bar{\chi}^i\Gamma_M\Gamma_7\epsilon_j\right),\\
\delta_Q B_{MN} &\equiv -i\,\mathrm{e}^{-2\sqrt{\frac{1}{2}}\phi}\left(\bar{\psi}^i_{[M}\Gamma_{N]}\Gamma_7\epsilon_i-i\frac{1}{2}\bar{\chi}^i\Gamma_M\Gamma_7\epsilon_i\right),\\
\delta_Q\phi &\equiv \frac{1}{\sqrt{2}}\bar{\chi}^i\epsilon_i,\\
\delta_Q\psi_{Mi} &\equiv \mathcal{D}_M\epsilon_i-i\frac{1}{8\sqrt{2}}\left(g\,\mathrm{e}^{\sqrt{\frac{1}{2}}\phi}+m\,\mathrm{e}^{-3\sqrt{\frac{1}{2}}\phi}\right)\Gamma_M\Gamma_7\epsilon_i\\
&\quad +i\frac{1}{8\sqrt{2}}\,\mathrm{e}^{-\sqrt{\frac{1}{2}}\phi}\left(\Gamma_M{}^{PQ}-6\delta^P_M\Gamma^Q\right)\left(mB_{PQ}\delta^j_i-i\Gamma_7F_{PQ}{}^I(\sigma^I)_i{}^j\right)\epsilon_j\\
&\quad +\frac{1}{8}\,\mathrm{e}^{2\sqrt{\frac{1}{2}}\phi}\Gamma_7\Gamma^{PQR}\partial_{[P}B_{QR]}\epsilon_i,\\
\delta_Q\chi_i &\equiv -i\sqrt{\frac{1}{2}}\Gamma^M(\partial_M\phi)\epsilon_i+\frac{1}{4\sqrt{2}}\left(g\,\mathrm{e}^{\sqrt{\frac{1}{2}}\phi}-3m\,\mathrm{e}^{-3\sqrt{\frac{1}{2}}\phi}\right)\Gamma_7\epsilon_i\\
&\quad +i\frac{1}{4}\,\mathrm{e}^{2\sqrt{\frac{1}{2}}\phi}\Gamma_7\Gamma^{PQR}\partial_{[P}B_{QR]}\epsilon_i\\
&\quad -\frac{1}{4\sqrt{2}}\,\mathrm{e}^{-\sqrt{\frac{1}{2}}\phi}\Gamma^{PQ}\left(mB_{PQ}\delta^j_i-i\Gamma_7F_{PQ}{}^I(\sigma^I)_i{}^j\right)\epsilon_j.
\end{aligned} \tag{7}
$$

Here $\epsilon_i (i=1,2)$ are the parameters of supertransformations, which satisfy the symplectic Majorana condition. Thus the theory is $N=(2,2)$ D=6 AdS supergravity with the gauge group SU(2) (having 16 supercharges). Note that we count the

number of supersymmetry by the number of supertransformation parameters. The commutator of two supersymmetry transformations satisfies

$$[\delta_{Q_1}(\epsilon_1), \delta_{Q_2}(\epsilon_2)] = \delta_{\rm G}(\xi^M) + \delta_{\rm L}(\Sigma^{AB}) + \delta_{\rm SU(2)}(\Lambda^I), \tag{8}$$

where the parameters of the general coordinate transformation ξ^M, the local Lorentz transformation Σ^{AB} and the SU(2) gauge transformation Λ^I are defined respectively as,

$$\begin{aligned}
\xi^M &= -i\bar{\epsilon}_2^i \Gamma^M \epsilon_{1i}, \\
\Sigma^{AB} &= -\xi^M {\omega_M}^{AB} \\
&\quad + \frac{1}{4\sqrt{2}} \,{\rm e}^{-\sqrt{\frac{1}{2}}\phi} \bar{\epsilon}_2^i \left({\Gamma^{AB}}_{CD} + 6\delta^{AB}_{CD} \right) \left(m B_{PQ} \delta_i^j - i\Gamma_7 F^I_{PQ} {(\sigma^I)_i}^j \right) e^{PC} e^{QD} \epsilon_{1j} \\
&\quad - \frac{1}{4} \bar{\epsilon}_2^i \left[{\rm e}^{2\sqrt{\frac{1}{2}}\phi} \left({\Gamma^{AB}}_{CDE} + 6\delta^A_C \delta^B_D \Gamma_E \right) \partial^{[P} B^{QR]} e^{PC} e^{QD} {e_R}^E \right. \\
&\quad \left. + \frac{1}{\sqrt{2}} \left(g\,{\rm e}^{\sqrt{\frac{1}{2}}\phi} + m\,{\rm e}^{-3\sqrt{\frac{1}{2}}\phi} \right) \Gamma^{AB} \right] \epsilon_{1i}, \\
\Lambda^I &= \xi^M {A_M}^I + i\frac{1}{\sqrt{2}} {(\sigma^I)_i}^j \bar{\epsilon}_2^i \Gamma_7 \epsilon_{1j}.
\end{aligned} \tag{9}$$

In order to calculate the r-dependence for the fields and the transformation parameters, we need to expand the fields around the vacuum expectation value of the scalar field ϕ. There are two stationary points:

$$\phi = \frac{1}{2\sqrt{2}} \ln \frac{m}{g}, \qquad \frac{1}{2\sqrt{2}} \ln \frac{3m}{g}. \tag{10}$$

We can find that the former is not supersymmetric whereas the latter preserves the supersymmetry as is known by solving the Killing spinor equation. We choose the stable point ϕ_0 as the latter one

$$\phi_0 = \frac{1}{2\sqrt{2}} \ln \frac{3m}{g}. \tag{11}$$

At first sight, since this point is the maximum point of the potential, the supersymmetric vacuum does not seem to be stable. However, according to the extended version of Breitenlohner-Freedman stability condition [14], [15], we see that it should be stable.

3. The Gauge Fixing Conditions and the Boundary Behaviors for the Fields

In the following sections, we will construct the conformal supergravity transformations from the AdS supergravity. First, we introduce the gauge fixing conditions such that the direction r is normal to the boundary, and denote the other directions of the space as $\mu, \nu \cdots = 0, 1, \cdots, 4$. The local Lorentz indices, $a, b, \cdots$ runs among $0, 1, \cdots, 4$. We choose the AdS$_6$ metric as

$$g_{MN} dx^M dx^N = \frac{K^2}{r^2} \left[dr dr + \hat{g}_{\mu\nu} dx^\mu dx^\nu \right], \tag{12}$$

where $\hat{g}_{\mu\nu}$ is the arbitrary five dimensional metric and the cosmological constant is given by

$$\Lambda = K^{-2} = 6\sqrt{3} g^{\frac{3}{2}} m^{\frac{1}{2}}. \tag{13}$$

We choose the gauge satisfying

$$e_r{}^5 = \frac{K}{r}, \qquad e_r{}^a = e_\mu{}^5 = A^I_r = \psi_{ri} = 0, \tag{14}$$

for which the rest of the fields are arbitrary. Note that our choice is just an which leads to a successful result. Next, we calculate the r-dependence of the fields. Define the fluctuation $\varphi = \phi - \phi_0$, then we obtain the field equation for φ as follows:

$$r^6 \partial_r (r^{-4} \partial_r \varphi) + 6\varphi = 0, \tag{15}$$

where we omit the higher order terms in r. Then we see that r-dependence of φ is r^2 or r^3. It is enough to choose the dominant one $\varphi \sim r^2$, since we want to take the near boundary limit $r \to 0$. Similarly the linearized field equation for A^I_μ gives, $A^I_\mu \sim r^0$.

As for B_{MN}, consider first the r-dependence of $B^{\mu\nu}$ and $B^{\mu r}$ since we have $\mathcal{D}_\mu B^{r\rho}$ in the field equations. The field equations read:

$$3\mathcal{D}_M \mathcal{D}^{[M} B^{NP]} - 2K^{-2} B^{NP} = 0. \tag{16}$$

Multiplying $\mathcal{D}_N$ to this equation, we obtain

$$\mathcal{D}_N B^{NP} = 0. \tag{17}$$

Then we substitute it back to (16), and finally we get the field equation as

$$\{\mathcal{D}_M \mathcal{D}^M - 6K^{-2}\} B^{NP} = 0. \tag{18}$$

We have to know the order in r for these terms, so that we write down the part of the above equation explicitly:

$$\mathcal{D}_M B^{M\nu} = \partial_r(\sqrt{-g} g^{rr} g^{\nu\rho} B_{r\rho}) + \partial_\mu(\sqrt{-g} g^{\mu\lambda} g^{\nu\rho} B_{\lambda\rho}), \tag{19}$$

where $B_{r\rho}$ gives the higher order in r of $B_{\lambda\rho}$, since these two terms in (19) have the same power in r. Thus, we can treat $\mathcal{D}_\mu B^{r\rho} = -\frac{1}{r}\hat{g}_{\mu\nu} B^{\nu\rho} +$ higher order, so that we cannot neglect $B_{r\rho}$ term. The linearized field equations $B^{\mu\nu}$ are now

$$r^2 \partial_r^2 B^{\nu\rho} - 8r B^{\nu\rho} + 18 B^{\nu\rho} = 0. \tag{20}$$

Thus finally we obtain the behavior of the antisymmetric tensor field as $B_{\mu\nu} \sim r^{-1}$.

In the case of fermionic fields, we introduce $\tilde{\Gamma} \equiv \Gamma^5 \Gamma_7$ and define the projection operators $P_\pm = \frac{1}{2}(1 \pm \tilde{\Gamma})$. They play a role of 'chiral projection on operators' in the odd dimensional AdS supergravity. For even dimensional AdS supergravity, we can define the chirality, whereas we cannot in odd dimensions. We assign the positive eigenvalue of $\tilde{\gamma}$ to the upper component ψ_+, and the negative one to the lower component ψ_-.

We multiply $P_\pm$ to the field equations of χ_i, and we have

$$r\Gamma^5 \partial_r \chi_{i\mp} + r\Gamma^\mu \partial_\mu \chi_{i\pm} + \frac{5}{2}\Gamma^5 \chi_{i\mp} + \Gamma_7 \chi_{i\mp} = 0. \tag{21}$$

First, to know the main behavior, we omit the second term which is a higher order one. By multiplying Γ^5 from the left hand side, we obtain

$$\begin{aligned} r\partial_r \chi_{i+} + \frac{5}{2}\chi_{i+} + \chi_{i+} &= 0, \\ r\partial_r \chi_{i-} + \frac{5}{2}\chi_{i-} - \chi_{i-} &= 0. \end{aligned} \tag{22}$$

We find that $\chi_{i\pm}$ behave as

$$\chi_{i+} \sim r^{\frac{3}{2}}, \qquad \chi_{i-} \sim r^{\frac{7}{2}}. \tag{23}$$

Then we substitute these results into (21), we obtain two sets of solution $\chi_{i+} \sim r^{\frac{3}{2}}, \quad \chi_{i-} \sim r^{\frac{5}{2}}$ and $\chi_{i+} \sim r^{\frac{9}{2}}, \quad \chi_{i-} \sim r^{\frac{7}{2}}$. We choose the dominant behavior set

$$\chi_{i+} \sim r^{\frac{3}{2}}, \qquad \chi_{i-} \sim r^{\frac{5}{2}}. \tag{24}$$

Similarly, we obtain r-dependence of the Rarita-Schwinger fields as $\psi_{\mu i-} \sim r^{-\frac{1}{2}}$ and $\psi_{\mu i+} \sim r^{+\frac{1}{2}}$, respectively.

4. Local symmetries on the boundary

In what follows, using the supergravity fields whose r-dependence were obtained explicitly in the previous section to solve r-dependence of the local transformation parameters. We obtained in the last section the boundary behaviors of the fields as

$$\begin{aligned} e_\mu{}^a &= \left(\frac{r}{K}\right)^{-1} \hat{e_\mu{}^a}, \quad \psi_{\mu i+} = \left(\frac{r}{K}\right)^{\frac{1}{2}} \hat{\psi}_{\mu i+}, \quad \psi_{\mu i-} = \left(\frac{r}{K}\right)^{-\frac{1}{2}} \hat{\psi}_{\mu i-}, \\ B_{\mu\nu} &= \left(\frac{r}{K}\right)^{-1} \hat{B}_{\mu\nu}, \quad B_{\mu r} = \left(\frac{r}{K}\right)^{0} \hat{B}_{\mu r}, \quad A^I_\mu = \left(\frac{r}{K}\right)^{0} \hat{A}^I_\mu, \\ \chi_{i+} &= \left(\frac{r}{K}\right)^{\frac{3}{2}} \hat{\chi}_{i+}, \quad \chi_{i-} = \left(\frac{r}{K}\right)^{\frac{5}{2}} \hat{\chi}_{i-}, \quad \varphi = \left(\frac{r}{K}\right)^{2} \hat{\varphi}. \end{aligned} \tag{25}$$

Here the hatted fields become arbitrary functions of x^μ, being independent of r on the boundary.

We substitute (25) and the gauge fixing conditions (12) into the transformation law in six dimensions. To keep the gauge fixing conditions, we need some constraints on the parameters, whose solution represents the 'residual symmetry'. For example, from the transformation law for the $e_r{}^5$, we read

$$\delta\hat{e}_r{}^5 = \xi^r \partial_r \hat{e}_r{}^5 + \xi^\nu \partial_\nu \hat{e}_r{}^5 + \partial_r \xi^r \hat{e}_r{}^5 + \partial_r \xi^\nu \hat{e}_\nu{}^5 + \Sigma^5{}_a \hat{e}_r{}^a + \hat{\bar{\psi}}_r^i \gamma^5 \epsilon_i, \tag{26}$$

which should vanish because of the gauge fixing. Then we have

$$\partial_r \left(\frac{1}{r} \xi^r \right) = 0, \tag{27}$$

and its solution is $\xi^r \sim r$. We can finally obtain r-dependence for the transformation parameters as

$$\begin{aligned} \epsilon^{\pm} &= \left(\frac{r}{K} \right)^{\pm\frac{1}{2}} \hat{\epsilon}^{\pm}, \quad \xi^r = \left(\frac{r}{K} \right)^{1} \hat{\xi}^r, \quad \xi^\nu = \left(\frac{r}{K} \right)^{0} \hat{\xi}^\nu, \\ \Lambda^I &= \left(\frac{r}{K} \right)^{2} \hat{\Lambda}^I, \quad \Sigma^{ab} = \left(\frac{r}{K} \right)^{0} \hat{\Sigma}^{ab}, \quad \Sigma^{a5} = \left(\frac{r}{K} \right)^{1} \hat{\Sigma}^{a5}, \end{aligned} \tag{28}$$

where the hatted functions mean the one of the boundary coordinate x^μ and do not depend on the r-direction, as before.

Before substituting (25) and (28) into the local transformations of the six dimensional AdS supergravity (7), we have to examine the fields which are not independent, and express them by the other fields. To do this, we use the equations of motion for the fields including the interaction terms but ignoring 2-fermi terms, since they contribute to the higher order fermi terms. Then we pull out the dominant contributions having the lowest power of r by taking the near boundary limit, and finally express the field in terms of the independent fields. As for the fermion fields, we deal with the fields having the higher order powers in r. Now we express the fields which is obtained by taking the $r \to 0$ limit as the ones with suffix 0. We have

$$B_{0\mu r} = \frac{1}{\sqrt{2}m} \left(\frac{3m}{g} \right)^{\frac{3}{4}} \mathcal{D}_{0\nu} B^{\nu}_{0\,\mu} + \frac{1}{4} \left(\frac{3m}{g} \right)^{\frac{1}{2}} e_0^{-1} \epsilon_{\mu\nu\rho\sigma\tau} B_0^{\nu\rho} B_0^{\sigma\tau}. \tag{29}$$

As for $\psi_{0\mu i-}$, we apply the projection operator P_- onto the field equation. Then we find the lowest order is $\mathcal{O}(r^{\frac{5}{2}})$, and we obtain

$$-\frac{3}{K} \Gamma_0^{\mu\nu} \Gamma^5 \psi_{0\nu i+} = \Gamma_0^{\mu\nu\rho} \mathcal{D}_{0\nu} \psi_{0\rho i-} - \frac{3m}{4\sqrt{2}} \left(\frac{3m}{g} \right)^{-\frac{1}{4}} B_0^{\rho\sigma} \Gamma_0^{[\mu} \Gamma_{0\sigma\rho} \Gamma_0^{\nu]} \psi_{0\nu i-}. \tag{30}$$

Let us define ϕ^μ_{0i-} by

$$\phi^\mu_{0i-} \equiv \Gamma_0^{\mu\nu\rho} \mathcal{D}_{0\nu} \psi_{0\rho i-} - \frac{3m}{4\sqrt{2}} \left(\frac{3m}{g} \right)^{-\frac{1}{4}} B_0^{\rho\sigma} \Gamma_0^{[\mu} \Gamma_{0\sigma\rho} \Gamma_0^{\nu]} \psi_{0\nu i-}. \tag{31}$$

Then, solving (30) we can express $\psi_{0\mu i+}$ only in terms of independent fields, namely,

$$\psi_{0\mu i+} = -\frac{K}{3}\Gamma^5\left(g_{0\mu\nu} - \frac{1}{4}\Gamma_{0\mu}\Gamma_{0\nu}\right)\phi^{\nu}_{i-}. \tag{32}$$

For χ_{0i+}, we use the projection P_+, and from the lowest order terms of $\mathcal{O}(r^{\frac{5}{2}})$, we can obtain

$$\begin{aligned}
-\frac{1}{K}\Gamma_7\chi_{0i-} &= \Gamma^{\mu}_0\mathcal{D}_{0\mu}\chi_{0i+} - \frac{m}{2\sqrt{2}}\left(\frac{3m}{g}\right)^{-\frac{1}{4}}\underline{B_{0\mu r}}\Gamma^{\rho}_0\Gamma^{\mu}_0\Gamma^5\psi_{0\rho i-} \\
&\quad - \frac{m}{2\sqrt{2}}\left(\frac{3m}{g}\right)^{-\frac{1}{4}}B_{0\mu\nu}\Gamma^{\rho}_0\Gamma^{\mu\nu}_0\underline{\psi_{0\rho i+}} \\
&\quad + \frac{1}{4\sqrt{2}}i\left(\frac{3m}{g}\right)^{-\frac{1}{4}}F_{0\mu\nu}\Gamma^{\rho}_0\Gamma^{\mu\nu}_0\Gamma_7(\sigma^I)_i{}^j\psi_{0\rho j-} - \frac{m}{2\sqrt{2}}\left(\frac{3m}{g}\right)^{-\frac{1}{4}}B_{0\mu\nu}\Gamma^{\mu\nu}_0\chi_{0i+} \\
&\quad - \frac{1}{4}\left(\frac{3m}{g}\right)^{-\frac{1}{4}}\partial_\mu B_{0\nu\rho}\Gamma^{\sigma}_0\Gamma^{\mu\nu\rho}_0\Gamma_7\psi_{0\sigma i-} + \frac{1}{\sqrt{2}K}\varphi_0\Gamma^{\mu}_0\Gamma^5\psi_{0\mu i-}.
\end{aligned} \tag{33}$$

The underlined fields are expressed by (29) and (32).

Furthermore, we decompose 16 component spinors ψ into two eight component ones, since in five dimensions, the number of components of a spinor is eight. We decompose ψ as

$$\begin{aligned}
\psi_{\pm} &= \frac{1}{2}(1 \pm \tilde{\Gamma})\psi, \\
\psi_- &= \begin{pmatrix} 0 \\ \psi^{\mathrm{D}} \end{pmatrix}, \quad \psi_+ = \begin{pmatrix} \psi^{\mathrm{U}} \\ 0 \end{pmatrix}.
\end{aligned} \tag{34}$$

Here, we have represented the gamma matrices as

$$\begin{aligned}
\Gamma^a &= \gamma^a \otimes \sigma^3, \\
\Gamma^5 &= \mathbf{1} \otimes \sigma^1, \\
\Gamma^7 &= \mathbf{1} \otimes (-i\sigma^2),
\end{aligned} \tag{35}$$

so that $\tilde{\Gamma} = \mathbf{1} \otimes \sigma^3$.

We redefine some of the fields to simplify the equations. For the antisymmetric tensor $B_{\mu\nu}$, to eliminate the derivative of ϵ_{0i-} in the supertransformation, we define

$$\tilde{B}_{0\mu\nu} \equiv B_{0\mu\nu} + \frac{1}{\sqrt{2}m}\left(\frac{3m}{g}\right)^{\frac{1}{4}}\bar{\psi}^i_{0[\mu}\psi_{0\nu]i}. \tag{36}$$

We also rescale the fields as

$$
\begin{aligned}
&\psi_{0\mu i+} \to K\psi_{0\mu i+}, \quad B_{0\mu\nu} \to \frac{\sqrt{2}}{m}\left(\frac{3m}{g}\right)^{\frac{1}{4}} B_{0\mu\nu}, \quad B_{0\mu r} \to \frac{3}{mg} B_{0\mu r} \\
&A^I_{0\mu} \to \frac{1}{g} A^I_{0\mu}, \quad \chi_{0i+} \to K\chi_{0i+}, \quad \chi_{0i-} \to K^2\chi_{0i-}, \quad \varphi_0 \to K^2\varphi_0,
\end{aligned} \tag{37}
$$

such that the transformations become independent of m and g. Also let define $\epsilon_{0i+} = -K\Gamma_7\eta_{0i-}$, where the η_{0i-} is an arbitrary function of x^μ.

Then, the definitions of various quantities become

$$
\begin{aligned}
D_{0\mu}\epsilon^{\mathrm{D}}_{0i} &= \left(\partial_\mu + \frac{1}{4}\omega_{0\mu}{}^{ab}\gamma_{ab}\right)\epsilon^{\mathrm{D}}_{0i} - i\frac{1}{2}A^I_{0\mu}(\sigma^I)_i{}^j\epsilon^{\mathrm{D}}_{0j}, \\
F^I_{0\mu\nu} &= 2\partial_{[\mu}A^I_{0\nu]} + \epsilon^{IJK}A^J_{0\mu}A^K_{0\nu}, \\
\tilde{B}_{0\mu\nu} &= B_{0\mu\nu} - \frac{1}{2}\bar{\psi}^{\mathrm{D}i}_{0[\mu}\psi^{\mathrm{D}}_{0\nu]i}, \\
\phi^{\mathrm{D}\mu}_{0i} &= -i\gamma^{\mu\nu\rho}\mathcal{D}_{0\nu}\psi^{\mathrm{D}}_{0\rho i} - \frac{3}{4}B^{\rho\sigma}_0\gamma^{[\mu}\gamma_{\rho\sigma}\gamma^{\nu]}\psi^{\mathrm{D}}_{0\rho i}.
\end{aligned} \tag{38}
$$

The expressions of the dependent fields become

$$
\begin{aligned}
\psi^{\mathrm{U}}_{0\mu i} &= -i\frac{1}{3}\left(g_{0\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu\right)\phi^{\mathrm{D}\nu}_{0i}, \\
B_{0\mu r} &= \mathcal{D}_{0\nu}\tilde{B}^\nu_{0\mu} + \frac{1}{2}e^{\mu\nu\rho\sigma\tau}\tilde{B}_{0\nu\rho}\tilde{B}_{0\sigma\tau}, \\
\chi^{\mathrm{D}}_{0i} &= -i\gamma^\mu\mathcal{D}_{0\mu}\chi^{\mathrm{U}}_{0i} - i\frac{1}{4}\underline{B_{0\mu r}}\gamma^\rho\gamma^\mu\psi^{\mathrm{D}}_{0\rho i} - i\frac{1}{2}\tilde{B}_{0\mu\nu}\gamma^\rho\gamma^{\mu\nu}\psi^{\mathrm{D}}_{0\rho i} \\
&\quad + \frac{1}{24}F_{0\mu\nu}\gamma^{\mu\nu}\chi^{\mathrm{U}}_{0i} + \frac{1}{4}\partial_\mu\tilde{B}_{0\nu\rho}\gamma^\sigma\gamma^{\mu\nu\rho}\psi^{\mathrm{D}}_{0\sigma i} - \frac{1}{\sqrt{2}}\varphi_0\gamma^\mu\psi^{\mathrm{D}}_{0\mu i}.
\end{aligned} \tag{39}
$$

The underlined part in the third expression is written by the second one.

Taking $r \to 0$ limit, we obtain the local transformations on the boundary

$$
\begin{aligned}
\delta e_{0\mu}{}^a &= \xi^\nu_0\partial_{0\nu}e_{0\mu}{}^a + \partial_{0\mu}\xi^\nu_0 e_{0\nu}{}^a + \Omega_0 e_{0\mu}{}^a + \Sigma^a_{0b}e_{0\mu}{}^b - i\bar{\psi}^{\mathrm{D}i}_{0\mu}\gamma^a\epsilon^{\mathrm{D}}_{0i}, \\
\delta A^I_{0\mu} &= \xi^\nu_0\partial_{0\nu}A^I_{0\mu} + \partial_{0\mu}\xi^\nu_0 A^I_{0\nu} + \mathcal{D}_{0\mu}\Lambda^I_0 \\
&\quad -\frac{3}{2}\bar{\chi}^{i\mathrm{U}}_0(\sigma^I)_i{}^j\epsilon^{\mathrm{D}}_{0j} - 3i\underline{\bar{\psi}^{\mathrm{U}i}_{0\mu}}(\sigma^I)_i{}^j\epsilon^{\mathrm{D}}_{0j} - 3i\bar{\psi}^{i\mathrm{D}}_{0\mu}(\sigma^I)_i{}^j\eta^{\mathrm{D}}_{0j}, \\
\delta\varphi_0 &= \xi^\mu_0\partial_\mu\varphi_0 - 2\Lambda_0\varphi_0 + \frac{1}{\sqrt{2}}(-\underline{\bar{\chi}^{\mathrm{D}i}_0}\epsilon^{\mathrm{D}}_{0i} + \bar{\chi}^{\mathrm{U}i}_0\eta^{\mathrm{D}}_{0i}), \\
\delta\tilde{B}_{0\mu\nu} &= \xi^\rho_0\partial_\rho\tilde{B}_{0\mu\nu} + \partial_\mu\xi^\rho_0\tilde{B}_{0\rho\nu} + \partial_\nu\xi^\rho\tilde{B}_{0\mu\rho} + \Omega_0\tilde{B}_{0\mu\nu}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}\bar{\chi}_0^{\mathrm{U}i}\gamma_{\mu\nu}\epsilon_{0i}^{\mathrm{D}}+\mathcal{D}_{0[\mu}\gamma_{\nu]}\epsilon_{0i}^{\mathrm{D}}+\underline{\bar{\psi}_{0[\mu}^{\mathrm{U}i}}\gamma_{\nu]}\epsilon_{0i}^{\mathrm{D}}+i\frac{1}{4}\bar{\psi}_{0[\nu}^{\mathrm{D}i}(\gamma_{\mu]}{}^{\rho\sigma}-4\delta_{\mu]}^{\rho}\gamma^{\sigma})\epsilon_{0i}^{\mathrm{D}}\tilde{B}_{0\rho\sigma},\\
\delta\psi_{0\mu i}^{\mathrm{D}} = {}&\xi_0^{\nu}\partial_{\nu}\psi_{0\mu i}^{\mathrm{D}}+\partial_{\mu}\xi_0^{\nu}\psi_{0\nu i}^{\mathrm{D}}+\frac{1}{2}\Omega_0\psi_{0\mu i}^{\mathrm{D}}+\frac{1}{4}\Sigma^{0ab}\gamma_{ab}\psi_{0\mu}^{\mathrm{D}i}-i\frac{1}{2}g\Lambda_0^I(\sigma^I)_i{}^j\psi_{0\mu j}^{\mathrm{D}}\\
&+\mathcal{D}_{0\mu}\epsilon_{0i}^{\mathrm{D}}-i\frac{1}{4}\tilde{B}_{0\nu\rho}(\gamma_{\mu}{}^{\nu\rho}-4\delta_{\mu}^{\nu}\gamma^{\rho})\epsilon_{0i}^{\mathrm{D}}-i\gamma_{\mu}\eta_{0i}^{\mathrm{D}},\\
\delta\chi_{0i}^{\mathrm{U}} = {}&\xi_0^{\mu}\partial_{\mu}\chi_{0i}^{\mathrm{U}}-\frac{3}{2}\Omega_0\chi_{0i}^{\mathrm{U}}-\frac{1}{4}\Sigma_0^{ab}\gamma_{ab}\chi_{0i}^{\mathrm{U}}-i\frac{1}{2}g\Lambda_0^I(\sigma^I)_i{}^j\chi_{0j}^{\mathrm{U}}\\
&-\frac{1}{\sqrt{2}}\varphi_0\epsilon_0^{\mathrm{D}i}-i\frac{1}{4}\gamma^{\mu\nu\rho}\partial_{[\mu}\tilde{B}_{0\nu\rho]}\epsilon_{0i}^{\mathrm{D}}-i\frac{1}{24}F_{0\mu\nu}^I\gamma^{\mu\nu}(\sigma^I)_i{}^j\epsilon_j^{\mathrm{D}}\\
&+\frac{1}{4}\underline{B_{0r\nu}}\epsilon_{0i}^{\mathrm{D}}-\frac{1}{2}\tilde{B}_{0\mu\nu}\gamma^{\mu\nu}\eta_{0j}^{\mathrm{D}}.
\end{aligned}
\tag{40}
$$

From these equations, we find Ω_0 is the Weyl transformation parameter, and η_{0i}^{D} that of the super Weyl transformation. The underlined terms are expressed by (39). We can count the off-shell degrees of freedom of the above fields:

$$
\begin{aligned}
&\mathrm{d.o.f}(\mathrm{e}_{0\mu}{}^{\mathrm{a}})=9,\quad \mathrm{d.o.f}(\mathrm{A}_{0\mu}^{\mathrm{I}})=12,\quad \mathrm{d.o.f}(\varphi_0)=1,\quad \mathrm{d.o.f}(\tilde{\mathrm{B}}_{0\mu\nu})=10,\\
&\mathrm{d.o.f}(\psi_{0\mu\mathrm{i}}^{\mathrm{D}})=24,\quad \mathrm{d.o.f}(\chi_{0\mathrm{i}}^{\mathrm{U}})=8.
\end{aligned}
\tag{41}
$$

We see that the bosonic degrees of freedom and the fermionic one are the same.

5. Summary and Discussions

We have attempted to construct a certain five-dimensional CSG. As a first step of this attempt, the local transformation laws of this CSG are determined. In general, the coefficients of the Weyl transformation parameter Ω_0 should be the Weyl weights, so that we could predict them in the five dimensional conformal supergravity.

A few interesting problems have remained. Although it is rather tedious work, we can construct the CSG in the standard fashion [17], [18], [19], and compare the result with that obtained in this paper. The construction would be very similar to that of the six dimensional CSG [16] since the gauge group in both theories is the same SU(2).

Furthermore, our result would give a key to understand the field theory of D4-D8 system [20], [10], [13]. Actually, our CSG field contents in five dimensions represent the pure gravity parts of the fields in the result of D4-D8 system [13].

If we execute the same method [8] using these CSG transformations, we will get the eta invariants instead of the anomalies of local symmetries*.

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References

[1] J. Maldacena, The large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* **2** (1998) 231, hep-th/9711200.

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Gauge theory correlators from noncritical string theory, *Phys. Lett.* **B428** (1998) 105, hep-th/9802109.

[3] E. Witten, Anti de Sitter space and holography, *Adv. Theor. Math. Phys.* **2** (1998) 253, hep-th/9802150.

[4] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Large N Field Theories, String Theory and Gravity, hep-th/9905111.

[5] H. Liu and A.A. Tseytlin, $D = 4$ super Yang-Mills, $D = 5$ gauged supergravity and $D = 4$ conformal supergravity, *Nucl. Phys.* **B533** (1998) 88, hep-th/9804083.

[6] M. Nishimura and Y. Tanii, Supersymmetry in the AdS/CFT correspondence, *Phys. Lett.* **B466** (1999) 37, hep-th/9810148.

[7] M. Nishimura and Y. Tanii, Local Symmetries in the $\mathrm{AdS}_7/\mathrm{CFT}_6$ Correspondence, *Mod. Phys. Lett.* **A14** (1999) 2709, hep-th/9910192.

[8] M. Nishimura and Y. Tanii, Super Weyl anomalies in the AdS/CFT correspondence, *Int. J. Mod. Phys.* **A** (1999) 3731, hep-th/9904010.

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[9] L.J. Romans, The F(4) Gauged Supergravity In Six-Dimensions, *Nucl. Phys.* **B269** (1986) 691.

[10] A. Brandhuber and Y. Oz, The D4-D8 Brane System and Five Dimensional Fixed Points, *Phys. Lett.* **B460** (1999) 307, hep-th/9905148.

[11] D. Youm, Partially Localized Intersecting BPS Branes, hep-th/9902208.

[12] M. Cvetic, H. Lu and C. N. Pope, Gauged Six-dimensional Supergravity from Massive Type IIA, *Phys. Rev. Lett.* **83** (1999) 5226, hep-th/9906221.

[13] S. Ferrara, A. Kehagias, H. Partouche and A. Zaffaroni, AdS_6 Interpretation of 5d Superconformal Field Theories, *Phys. Lett.* **B431** (1998) 57, hep-th/9804006.

[14] P. Breitenlohner and D. Z. Freedman, Stability in Gauged Extended Supergravity, *Ann. Phys.* **144** (1982) 249.

[15] L. Mezincescu and P. K. Townsend, Stability at a Local Maximum in Higher Dimensional Anti-de Sitter Space and Applications to Supergravity *Ann. Phys.* **160** (1985) 406

[16] E. Bergshoeff, E. Sezgin and A. van Proeyen, Superconformal Tensor Calculus and Matter Couplings in Six-dimensions, *Nucl. Phys.* **B264** (1986) 653.

[17] M. Kaku, P.Townsend and van P. van Nieuwenhuizen, Properties of conformal supergravity, *Phys. Rev.* **D17** (1978) 3179

[18] P. van Nieuwenhuizen, Supergravity, *Phys. Rep.* **68** (1981) 189.

[19] A. Salam and E. Sezgin, *Supergravities in Diverse Dimensions* (North-Holland/World Scientific, 1989).

[20] N. Seiberg, Five Dimensional SUSY Field Theories, Non-trivial Fixed Points and String Dynamics, *Phys. Lett.* **B388** (1996) 753, hep-th/9608111.